Paper 2 Math AA HL:

Question 1:

$f(x) = 1 - x^2$

$g(x) = e^{2x}$

a) Find the two intercepts of the two functions.
b) Find the area enclosed between the two functions.

ANSWER:

a) a = -0.917, b = 0
b) 0.240 with GDC

Question 2:

X	5	6	6	8	irrelevant
Y	9	13	p	q	21

The regression line for this function is $y = 2.1875x + 0.6875$.

a) Show that (7, 16) is the mean of this function.
b) Given that $q - p = 3$, find the values for p and q.

ANSWER:

a) Literally just substitute 7 into the x for the function and show it's equal to 16.
b) The mean of the y values is 16, so the sum of all the y values is 80. Since 9 + 13 + 21 is 43, $p + q$ is equal to 37. Since $q - p = 3$, $p = 17$ and $q = 20$.

Question 3:

The “loudness” of a sound is represented mathematically by the following function.

$L = 10 \times log_{10}(I \times 10^{12})$.

Sound 1 has intensity 10^{-6} units and is 60 decibels.

Sound 2 has twice the intensity of Sound 1.

a) What is the intensity of Sound 2?
b) Hence, what is the loudness of Sound 2?

A thunder strike has a loudness of 115 decibels.

c) Find the intensity of the thunderstrike.

ANSWER:

a) Obviously 2×10^{-6}
b) Substitute the new value for intensity found in a) and just use the GDC, 63.01 decibels
c) $115 = 10 \times log_{10}(I \times 10^{12})$ so $\frac{10^{11.5}}{10^{12}} = I$ which means intensity is 0.316

Question 4:

The velocity of a particle is $1 + e^{-t} - e^{-sin(2t)}$, $0 \leq t \leq 2$

a) What is the velocity of the particle when t = 2?
b) What is the maximum velocity of the particle?
c) When the particle's direction changes, what is the acceleration?

ANSWER:

a) Just substitute. -0.996 (here should be 0.203, I think)
b) Graph the function and find the maximum value which is 1.18 when x = 0.406.
c) The velocity changes directions when t = 1.66. Derive the velocity of the particle to get $-e^{-t} - e^{-sin(2t)} \times 2cos(2t)$ which if you substitute 1.66 you get about 2.16

Question 5:

$X \sim B(n,\ 0.25)$ and $P(X \geq 1) > 0.99$.

Find n.

ANSWER:
$P(X \geq 1) > 0.99 = P(X = 0) < 0.01$, so then you use the binomial distribution function formula [THIS IS **NOT** IN THE FORMULA BOOKLET.] $nCo \times (0.25)^0 \times (0.75)^n < 0.01$. Since the first two are both equal to 1, use nSolve to find when $(0.75)^n = 0.01$, which gives you a value of 16.007. **DO NOT** assume this means it is 16. The answer is 17 because if you substitute 16 into the equation the inequality does not hold true.

Question 6:

The volume of a sphere is increasing at $5cm^3$ per second. Given that the current volume is $20cm^3$, what is the rate of change for the radius?

ANSWER:

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ and $V = \frac{4}{3}\pi r^3$ and $\frac{dV}{dr} = 4\pi r^2$. When the volume is $20cm^3$, the radius is about 1.68389. By substituting these values, $5cm^3 = 4\pi(1.684)^2 \times \frac{dr}{dt}$ and $\frac{dr}{dt} = 0.140cm^{-1}$

Question 7:

A function $y = 4 \times ln(x - 2)$ is rotated around the y-axis 360º from 0 ≤ x ≤ 4.

What is the volume of revolution for the function?

ANSWER:

Convert the function in terms of x. $\frac{y}{4} = ln(x - 2)$, then $e^{\frac{y}{4}} = x - 2$, then $e^{\frac{y}{4}} + 2 = x$, then $x^2 = e^{\frac{y}{2}} + 4$. Then, you take the numeric integral from 0 to 4 of that function. About 28.78 units.

Question 8:

I don't remember 8.

Question 9:

There is a function $\frac{x-4}{ax^2+bx+c}$ with y-intercept (2,1?) and vertical asymptote at x = 1.
Find a, b and c.

ANSWER:

Couldn't solve.

- SECTION B -

Question 10:

A chocolatier is selling chocolate in a store. The probability density function X is given as:

$f(x) = \frac{6}{85}(4 + 3x - x^2)$ for $0.5 \le x \le 3$.

$f(x) = 0$ for all other values.

a) What is the mode of the function?
b) What is $P(1 < x < 2)$?
c) What is the median of the function?

The store sells chocolate at $25 per kilogram, but it is running a promotion where if you buy more than 0.75 kilograms of chocolate, the rate becomes $24 per kilogram.

d) What is the probability that someone will purchase no more than $48 of chocolate?
e) What is the expected cost, to the nearest cent, of an average person at the store?

ANSWER:

a) Graphing the function gives the maximum (mode) at x = 1.5.
b) By setting a numeric integral with the bounds as 1 and 2, the probability is 0.435.
c) Using nSolve and setting the upper bound of the function as a, the median is 1.687.
d) $48 of chocolate is 1.5 kilograms. Do another bounded numeric integral to get 0.418.
e) Take the integral from 0.5 to 3 of the function times x to get the expected value of 1.704 kg. Since this is above 0.75, you multiply this value by 24 in order to get about $40.90.

Question 11:

A sprinkler sprays in a circular motion where the radius is 20m. There is a chord AB in the circle where the distance OA and OB are both 20m and the distance to the midpoint of AB is 14m.

a) Show that the length of AB is 28.57 to four significant figures.

The sprinkler makes one full revolution in 16 seconds.

b) Show that the angular rotation of the sprinkler is $\frac{\pi}{8}$ radians per second.

The time T is the time it takes for the sprinkler to go through all of AB.

c) Find T.

There is now another point D along chord AB that can move. The length AD is called d. The angle OAD is β and the angle AOD is α. When a certain time *t* = 0, the sprinkler is collinear with the point A. When the sprinkler is collinear with point D, time *t* = *t*.

d) Find an expression for α in terms of *t*.

β is given as 0.7754 radians.

e) Using the sine rule in triangle AOD, show that $d = \frac{20 \times sin(\frac{\pi t}{8})}{sin(2.37 - \frac{\pi t}{8})}$.

A frog is hopping along chord AB, and its distance from point A is given by the function:

$$g(t) = 0.05t^2 + 1.1t + 18$$

f) When t = 0, how far away is the frog from point A?

The variable *w* denotes the distance from the frog to point D.

g) Create an expression for *w* using $d(t)$ and $g(t)$.

h) When and where will the water and the frog meet?

ANSWER:

a) Using $a^2 + b^2 = c^2$, the distance to the midpoint is 14.2828. Multiply this by 2.

b) The sprinkler goes 2π in 16 seconds. Divide this to get $\frac{\pi}{8}$ easily.

c) Calculate the angle between point A and the midpoint. $cos^{-1}(\frac{14}{20}) = 0.7954$ radians. Multiply this angle by two and then divide by $\frac{\pi}{8}$ to get T = 4.051 seconds.

d) I'm pretty sure it's just $\frac{\pi t}{8}$. Like why else would part e) use $sin(\frac{\pi t}{8})$

e) $\frac{d}{sin(\alpha)} = \frac{20}{sin(\pi - \beta - \alpha)}$ then $d = \frac{20 \times sin(\frac{\pi t}{8})}{sin(3.1415 - 0.7754 - \frac{\pi t}{8})}$ then $d = \frac{20 \times sin(\frac{\pi t}{8})}{sin(2.37 - \frac{\pi t}{8})}$

f) Obviously 18 meters.

g) w is $g(t) - d(t)$ so just substitute, giving $0.05t^2 + 1.1t + 18 - \frac{20 \times sin(\frac{\pi t}{8})}{sin(2.37 - \frac{\pi t}{8})}$.

h) Graph *w* and the zero should be 2.29 seconds. Substitute this value to get 5.04 meters.

Question 12:

A differential equation is given as $\frac{dy}{dx} - ycosec(2x) = \sqrt{tan(x)}$. $y = \frac{\pi}{4}$ when $x = \frac{\pi}{4}$.

a) Using Euler's method with variable step $\frac{\pi}{12}$, find an approximation for $\frac{5\pi}{12}$.
b) Show that $\frac{dy}{dx}(\frac{1}{2}ln(cot(x))) =- 2cosec(2x)$
c) I forgot the rest.

ANSWER:

a) It takes miserably long but I think it's y = 1.986.
b) Also equally miserably long but things of note is to use the chain rule to derive the left hand side. Eventually you will need to split cosec and cot into $\frac{1}{sin(x)}$ and $\frac{cos(x)}{sin(x)}$.